

In a nutshell: The quadratic formula

Given a quadratic equation $ax^2 + bx + c = 0$ with real coefficients and $a \neq 0$, if we are finding two real roots and $b \neq 0$, then one root is larger in absolute value and the other smaller in absolute value.

To find the root that is larger in absolute value, use the formula as follows:

$$\text{If } b > 0, \text{ use } \frac{-b - \sqrt{b^2 - 4ac}}{2a}; \text{ otherwise } b < 0, \text{ so use } \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

To find the root that is smaller in absolute value, use the formula as follows:

$$\text{If } b > 0, \text{ use } \frac{-2c}{b + \sqrt{b^2 - 4ac}}; \text{ otherwise } b < 0, \text{ so use } \frac{-2c}{b - \sqrt{b^2 - 4ac}}.$$

In both cases, we add either two positive numbers or add two negative numbers, so as to avoid subtractive cancellation.